Synthese

(Author's finally accepted version of manuscript) The final publication is available at Springer via http://dx.doi.org/ [DOI :10.1007/s11229-015-0950-7]

Normativity, probability, and meta-vagueness

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Abstract:

This paper engages with a specific problem concerning the relationship between descriptive and normative claims. Namely, if we understand that descriptive claims frequently contain normative assertions, and vice versa, how then do we interpret the traditionally rigid distinction that is made between the two, as 'Hume's law' or Moore's 'naturalistic fallacy' argument offered. In particular, Kripke's interpretation of Wittgenstein's 'rule-following paradox' is specially focused upon in order to re-consider the rigid distinction. As such, the paper argues that if descriptive and normative claims are not mutually exclusive, then we need a new framework with which to understand this relationship. In this regard, the paper borrows from concerns with vagueness, particularly using a degree-theoretic approach in terms of subjective probability, in an attempt to graphically figure out these differences. Consequently, the paper tentatively proposes the hyperbola model in which degrees of normativity and degrees of descriptivity could be expressed and measured. It is hoped, as a result, that this tentative proposal will contribute to deepening the debate on vagueness in general.

Keywords:

Normativity, descriptivity, probability, vagueness, rule-following, Kripke

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1 Normativity matters

It is intuitively obvious that a matter of fact which happened to occur once has no implication for what might happen next. If I happened to sit down on a particular seat at a particular restaurant, for example, there is nothing that requires me to sit in the same seat the next time I go to that restaurant. However, if the same action were repeated to the point of accumulation, a habit or a tendency to repeat that action could arise and, under certain circumstances, the habit might urge us to continue repeating the same event or action. This seems to be inherent in human nature. For example, if I have sat in exactly the same seat at the same restaurant every day for a month, I feel obliged to maintain the custom. It's even possible that the owner of the restaurant might feel obliged to save the seat for me when I typically come.

This phenomenon is applicable to many territorial issues between countries concerning small islands. If some fishermen of a particular country happened to stop at an inhabited island dozens of times in the past, their country might claim a right of possession of the island, although other countries could claim the same right because their fishermen have also stopped at the island in the past. Of course, the answer to this dilemma relies on political or legal definitions of territory and prior occupation. However, the problem itself serves as another example of an initial, accidental matter of fact (stopping at the island) being repeated often enough such that it might become a kind of force that urges us to maintain the fact. In other words, a sentence describing a fact – 'We have occupied this island' – seems to give evidential support to a derivative normative claim – 'We ought to occupy this island'.

However, facts that can be described in an indicative manner cannot always give evidential support to the normative claims that may be derived from them. Rather, and as a result of experience, it may seem 'natural' to distinguish normative claims from descriptive facts, even if the facts are iterated repeatedly. Suppose, for instance, that a student is routinely beaten by a fellow student. Could we then justify the claim that the beating of fellow students ought to be permitted, based on the fact that it has occurred repeatedly in the past? Intuitively, we are inclined to say no; facts, even if they have occurred many times before, do not necessarily justify normative claims. This example corresponds with the traditional distinction between *is* and *ought to*, between descriptive and normative claims. David Hume underlined the distinction in a formulation often referred to as 'Hume's Law': 'Of a sudden I am surpriz'd to find, that instead of the usual copulations of propositions, *is*, and *is not*, I meet with no proposition that is not connected with an *ought*, or an *ought not* … this *ought*, or *ought not*, expresses some new relation or affirmation, 'tis necessary that it shou'd be observ'd and explain'd' (Hume 2000, T3.1.1.27). Following Hume's formulation is the 'naturalistic fallacy' argument, as presented by G. E. Moore (see Moore 1903) in his refutation of J. S. Mill's view that we can derive the normative 'desirable' from the descriptive 'desired' (Mill 1987, p. 307). The distinction is certainly also bolstered by modest commonsense. Even

if many people violate a traffic law as a matter of fact, it is imperative that we still ought to observe the law, just as a student ought not to be beaten regardless of whether the student has been beaten before.

Nonetheless, we hesitate to declare that the distinction between descriptive and normative claims always hold true. The previous example, concerning territorial issues, may validate this hesitation. There are, however, problematic cases which invoke the connection between descriptive facts and normative claims far more strongly. I'll mention two cases, both of which have been maintained for more than a thousand years. The first is the practice in many cultures of burying dead people in the ground. On the surface, this seems to be a mere matter of fact. However, many feel reluctant to abolish this tradition, as we, consciously or unconsciously, accept the normative sentence 'We ought to bury the dead' because of an overwhelmingly long accumulation of facts. The second case is concerned with the succession lineage of the Imperial throne in Japan. For the last 125 successions, the Japanese monarchy has been headed by the male line of the Imperial Family. Apart from a legal stipulation requiring this, the succession lineage can also be seen as a historical matter of fact. However, how *should* successions be managed in the future? For instance, could the current generation change the tradition such that a successor could be adopted from the female line, especially considering that the maintenance of the male line is but a historical fact? This is a controversial issue in Japan, generating numerous debates. However, it seems to me that for many Japanese people, the change in tradition is seen as somehow beyond their discretionary power. Can we assume that this generation has some particular authority, especially in comparison to other generations, that would allow them to impose a change? In contemplating such an act, one is frequently overwhelmed by the sheer weight of the historical facts. In this case, the descriptive fact, 'the Japanese Imperial Throne has continually been succeeded by the male line of the Imperial Family', seems to strongly imply a normative claim, 'the Japanese Imperial Throne *ought* to be succeeded by the male line of the Imperial Family', irrespective of the present legal concerns. As such, if we questioned *when* the descriptive fact became a reason for the normative claim, the example of the succession of the Imperial throne in Japan appears to be one case in which the distinction between descriptive and normative claims is not clear-cut.

2 Meta-vague predicates

This article's primary aim is to explore the possibility of a more appropriate model (than the dichotomous distinction) with which to understand the tangled relationship between descriptive and normative claims. It does so by outlining a hypothetical model that takes into account subjective probability. In developing such an argument, I will examine the classical rule-following paradox, or what has become known in contemporary philosophy as the Kripkenstein paradox, as a means of engaging with this relationship. The 'following the rule' phenomenon is employed as a central example because it is a useful means of highlighting the

relationship between descriptive and normative claims.

Before I do so, however, I would like to draw the reader's attention to two concerns. Firstly, while I represent normative claims as altogether having the same quality, I am fully aware that there are different forms of normativity, found, for instance, in the differences between logical, legal, moral concerns, and the Kantian transcendental type of normativity. In spite of this, I understand that there is a common characteristic that defines all of these normative positions, including the Kantian one. Namely, there is always some kind of sanction that may be imposed if one were to violate the 'ought to' dictum. This point will be further explored below.

Secondly, the argument will be developed presupposing that the complex relationship between descriptive and normative claims can be treated, ultimately, as a phenomenon of vagueness. Of course, in this instance, the claim is not the same as those problems usually associated with vague predicates, for example, 'child', 'red', 'tall', and so on. In all likelihood, 'descriptive' and 'normative' claims belong to a different *order* of predicates than the ones cited here. Indeed, as far as it can be said that the phrase 'something is red' is descriptive and that 'something is good' is normative, 'descriptive' and 'normative' can be characterised as examples of meta-vague predicates (even though the term 'meta-vague' is most appropriately applied to the meta-distinction between 'vague' and 'sharp'[1]). Even if this is the case, however, meta-vague predicates do share something intrinsic with the vagueness of other vague predicates – the predicates admit borderline cases. This requires clarification. For example, Hilary Putnam once argued there are 'concepts that defy simple classification as "descriptive or normative" – concepts like the concept of cruelty' (Putnam 2002, p. 24)[2]. The killing of a

1 By 'meta-vagueness' I mean the vagueness of predicates which are predicated of sentences rather than subjects. This corresponds to a semiotic distinction between object-language and meta-language. Suppose *A* to be a sentence (that may include a vague predicate like red, child, or good, and so on), and *P* to be a predicate. Thus, if '*A* is *P*' is vague, then the vagueness here could be called 'meta-vagueness', as this is the vagueness of the sentence. Please note that meta-vagueness should be differentiated from higher-order vagueness on the boundary between penumbra and definitely true/false cases. Regarding the vagueness of the predicate 'vague', Sorensen (1985) strongly suggests that any argument involving a vague predicate, including the predicate 'vague' itself, must be 'a self-refuting thesis' (Sorensen 1985, p. 136). I do not believe that the same pessimistic conclusion should be drawn in the case of a descriptive/normative distinction, as will be shown by my argument as a whole. In any case, the vagueness of 'vague' must be investigated further.

2 Richard Dietz, one of my colleagues, suggests that 'child' is more illuminating example than 'cruel' as having an inferential role both in descriptive reasoning and in normative reasoning. I fully agree that 'child' is another, maybe more helpful, example of this point. For the time being, however, I follow Putnam's example, 'cruel', in order to situate my argument

human that takes fourteen minutes, even under the auspices of a death penalty, is usually considered cruel[3]. However, is this affirmation simply concerned with descriptive facts? If we accept or reject this as cruel, do we, implicitly or explicitly, accept the further normative claim that we ought to continue or abolish the death penalty? Something like cruelty's ambivalence, that it can be both descriptive and normative, may also be reflected in the example of the territorial issues. It seems that when we recognise our fishermen's stopping at the inhabited island in the past dozens of times as a descriptive fact, we tend, implicitly or explicitly, to commit to a normative claim to support our right to its possession. These examples suggest that there are numerous instances in which the boundary between descriptive and normative claims is vague[4].

Moreover, a sorites paradox may also arise in analysing these meta-vague predicates. Continuing with the Japanese succession example, the present Emperor is the 125th on the throne, a product of 1500 years of history. Faced with such a long and indisputable history, the descriptive fact of male succession is likely to become normative. However, suppose that the present Emperor were only the 10th in a lineage stretching back a mere 150 years. Even this (less imposing) descriptive fact may be enough to compel some people to accept it as normative. However, what if we were to continually shorten the period of Imperial rule by increasingly incremental steps? Should it be accepted that the present Imperial system ought to be retained, even if it began only yesterday? At what point does a descriptive claim become a normative one? Is it possible to quantify this slippage? It is obvious that a sorites paradox can arise here, and we might formalize it as follows:

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more easily in the current debate on the relation between descriptivity and normativity. Actually, 'cruel' could play an inferential role in descriptive reasoning as well as in normative reasoning, in that, for example, a dog cruelly treated is damaged.

[3] In Japan where the death penalty (by hanging) is still retained under the constitutional restriction of not imposing cruel punishment, there has been the debate on whether killing by hanging is cruel or not. It takes about fourteen minutes to end a convict's life by hanging, but the court judged that it is not cruel by saying that convicts lose their consciousness instantly after being hanged. Of course, it is still a controversial issue.

[4] Dietz also raised a question about whether components of descriptivity and normativity in my examples only coexist as individual constituents (like different colours in one and the same picture) so that this does not lead to vagueness of the distinction between them. To this question, I will answer that descriptivity and normativity cannot separately coexist in the same sentence without being interrelated (unlike in the case of colours) because of their countervailing asymmetry which I will discuss later in section 7 by investigating how we should react if the relevant sentence conflicted with the matter of fact. Rather what I want to focus on is the relational fact that the more descriptive, the less normative, and vice versa.

(Pre1) ‘The Empire has ruled for 1 minute’ is purely descriptive, having no normative force.
(Pre2) ‘The Empire has ruled for 2 minutes’ is purely descriptive, having no normative force.
.
.
(Con) ‘The Empire has ruled for 1500 years’ is purely descriptive, having no normative force.[5]

The initial premises seem to be perfectly acceptable, while the conclusion is clearly unacceptable, at least from many Japanese people’s point of view. The differentiation at the beginning and end of the trajectory suggests that there are vague boundaries between ‘purely descriptive’ and ‘normative’. A structurally similar argument can also be built using the example of cruelty. Cutting a human being or an animal in two would undoubtedly be seen as cruel, and therefore we ought not to do it. In this case, the descriptive claim is directly linked to a normative one. However, what would happen if the organism that was to be cut in two was much smaller? Should we accept that the cutting in two of a microorganism ought not to be done? Perhaps it would be described as cruel by some, but we feel no need to derive a normative prescription from the action. However, following the supposition above, we would be required to admit such a normative claim – and the sorites paradox emerges again. With this in mind, it is hoped that the argument below will provide a new perspective on the debates concerning problems of vagueness in general, as well as the debate concerning the relationship between descriptive and normative claims more specifically.

3 Reengaging with the rule-following paradox

Re-examining the rule-following paradox, particularly as discussed by Kripke (1982), may provide the foundations for developing a new understanding of the problematic relationship between descriptive and normative claims. The rule-following paradox is, moreover, frequently described in relation to Goodman’s (1983) grue paradox, as both are supposed to share a similar structure. Outlining this common structure provides a useful entry point into the debate.

Both paradoxes emerge as a result of the tension between specific forms of *data*, and the *regularity* or iteration of that data. The relationship seems to be direct, and yet a

5 Of course, this formulation is made by simplifying the structure of the argument. Precisely speaking, the Imperial ruling is established according to various elements including historical and mythological contexts in addition to temporal length. However, this may not be a flaw in the argument, because such simplification is usually made in constructing the sorites paradox concerning typically vague predicates like ‘hot’ or ‘child’. What matters is whether the structure of the argument admits the sorites paradox or not.

one-to-one relationship cannot be established. Certainly both paradoxes seem to share a similar structure. Such a structural understanding can however cause us to overlook a crucial difference between them. There is, of course, a superficial difference, in that the one is concerned with meaning while the other with induction. However, meaning and induction actually complement each other. Induction can be carried out, and exists, through the meaning of the predicates used in the data set, while the meaning of words is invariably inferred inductively via their past usage and connotations. However, it seems to me that a further distinction is possible, one which requires a more substantive review of the Kripkenstein rule-following paradox. Wittgenstein's own argument will be temporarily ignored, as its analysis is beyond the limits of this study.

Kripke imagines that '68 + 57' is a computation that I have never performed, having only ever carried out addition with numbers less than 57. The supposition is perfectly possible. Answering correctly, I will obtain the answer '125'. It is correct, because 'plus', as the word has been intentionally used in the past, denotes a function which, when applied to the symbols I have called '68' and '57', yields the value '125'. The logic of addition has been procedurally implemented. However, a sceptical challenge could be posed. Namely, what if, when I used the term 'plus' in the past, the answer I intended to produce from the equation '68 + 57' was '5'? My intention that '68 + 57' should turn out to denote '125', based upon my use of the symbol '+', cannot rely on the fact that I explicitly gave myself instructions to produce the result of 125, because by the very hypothesis, I did no such thing. Certainly, I may think I ought to apply the same function or rule that I have applied so many times before. However, who is to say what function this is? I gave myself only a finite number of examples (all of which were smaller than 57) in instantiating this function (Kripke 1982, pp. 8–9). Perhaps in the past I used 'plus' and '+' to denote a function called 'quus', defined like this:

$$x \oplus y = x + y, \text{ if } x, y < 57$$
$$= 5 \text{ otherwise.}$$

Who is to say that this is not the function I previously meant by '+'? Consequently, if I answer '68 + 57 = 125', can I say I am merely applying a previously used rule, even though other possibilities may exist, such as 'quus'? As Kripke explains,

> How can I *justify* my present application of such a rule, when a sceptic could easily interpret it so as to yield any of an indefinite number of other results? It seems that my application of it is an *unjustified* stab in the dark. I apply the rule *blindly*. (Kripke 1982, p. 17. Emphasis added.)

Obviously, this remark shows that Kripke understands the problem to be a matter of justification. Consequently, the mentioning of any number of descriptive facts will not undermine the sceptical challenge, precisely because a justification is a normative claim. This

point is highlighted in his next remark:

> When I respond in one way rather than another to such a problem as '68+57', I can have no justification for one response rather than another … there is no fact about me that distinguishes between my meaning plus and my meaning quus. Indeed, there is no fact about me that distinguishes between my meaning a definite function by 'plus' … and my meaning nothing at all. (Kripke 1982, p. 21)

Following this view, Kripke utterly rejects the resulting 'dispositional account' as a misconception of the sceptic's problem. In this vein he notes,

> Suppose I do mean addition by '+'. What is the relation of this supposition to the question of how I will respond to the problem '68 + 57'? The dispositionalist gives a *descriptive* account of this relation: if '+' meant addition, then I will answer '125'. But this is not the proper account of the relation, which is *normative,* not descriptive. The point is not that, if I meant addition by '+', I *will* answer '125', but that, if I intend to accord with my past meaning '+', I *should* answer '125'. (Kripke 1982, p. 37)

It follows from this remark that Kripke's formulation ultimately depends upon a dichotomous distinction between descriptive fact and normative justification. At least in this context, the distinction works as a necessary condition for his arguments. Without the distinction, Kripke's argument does not work at all. In this sense, Wittgenstein's paradox, as interpreted by Kripke, is an extended variant of Moore's naturalistic fallacy argument. Consequently, if this is correct, we are unknowingly implicated in a naturalistic fallacy whenever we follow any rule solely because of custom or facts.[6]

[6] It was pointed out by an anonymous reviewer that the absence of a sharp distinction is not tantamount to the absence of a distinction – it is just a vague distinction. So why, the reviewer asks, is Kripke's formulation not compromised given the existence of a vague distinction? This observation is absolutely true. Actually, my aim is not to criticize Kripke's argument, but to elucidate the relation between two meta-predicates, that is, descriptivity and normativity. Therefore, if Kripke in reality admitted the vague boundary between two meta-predicates, my argument would result in bringing to light a hidden structure of Kripke's argument. For the time being, however, Kripke so obviously develops his argument at least in this context by appealing to the sharp contrast between descriptivity and normativity that I provisionally presuppose he uses the sharp contrast between two meta-predicates as a necessary condition for his argument.

4 Mutual permeability rather than penumbra

From a contemporary standpoint, how might one interpret Kripke's argument? Evidently, it does not seem convincing to adopt, along with Hume and Moore, a clear-cut dichotomy between descriptive and normative claims. Moreover, numerous arguments for the interconnectedness, rather than mutual exclusivity, of descriptive and normative claims have since been made. Using Putnam's example of 'cruelty' again:

> 'Cruel' simply ignores the supposed fact/value dichotomy and cheerfully allows itself to be used sometimes for a normative purpose and sometimes as a descriptive term … such concepts are often referred to as 'thick' ethical concepts. That the thick ethical concepts are counterexamples to the idea that there exists an absolute fact/value dichotomy has long been pointed out. (Putnam 2002, p. 35)

Needless to say, 'the absolute fact/value dichotomy' Putnam highlights is structurally the same as the division between descriptive and normative claims under discussion. Considering this, we may understand the Kripkenstein argument as critically problematic, as this formulation of the rule-following paradox depends, entirely or at least crucially, on a clear-cut dichotomy between descriptive and normative claims.

Equally however, a complete abandonment of the distinction does not seem reasonable, the result of which would be a kind of bland monism. This is because, as was mentioned in the first section, it seems that there *are* instances in which there is a clear difference between descriptive and normative claims. That a student physically beat another is basically a matter of description and historical fact, whereas inflicting violence on someone is normatively laden (and problematic), irrespective of whether or not it has occurred before (although if we interpret the physical event of a beating by others as 'violence', a similar normative concern to that of 'cruelty' arises).

It follows that there may exist *degrees* of difference between descriptive and normative claims, something Kripke's rule-following paradox might not explicitly take into account. Indeed, the notion of a 'degree' originates from the meta-vague distinction between descriptive and normative claims. In so doing, what kind or measurement of 'degree' may be introduced? In answering this, it is necessary to carefully re-examine the concepts of normativity and descriptivity that make up the Kripkenstein paradox.

Firstly, with regard to normativity, I begin with re-confirming that there is a distinction between the descriptive facts and normative claims that structures our actual lives. For instance, if I drive a car at 60km/h, thus violating a traffic law that stipulates that the speed limit is 50km/h, I *am* still driving at a speed of 60km/h, as a matter of descriptive fact. However, this fact does not negate the normative claim that I *ought to* drive at or slower than a speed of 50km/h. Here it might be said that there is a sharp distinction between what *is* and what *ought to* be. This is, however, not always the case.

Indeed, there may be numerous borderline cases, both practical and theoretical, in which the distinction between *is* and *ought to* can become vague. For instance, consider the following famous proposition, known as Hempel's raven (Hempel 1965, p. 12ff):

(i) All ravens are black.

In Hempel's paradox, proposition (i) is treated as a claim that can be empirically confirmed or falsified, resulting in it being regarded as a descriptive fact. However, apart from Hempel's argument concerning non-black, non-raven objects (a yellow banana, for example), yet another understanding is possible. How should we react, for instance, if we found a pink raven? One possible reaction to the discovery is to assert that proposition (i) has now been falsified, and that we must conclude that not all ravens are black. However, it is also plausible to say that the pink bird is not actually a raven at all, but a new species. In this instance, the property of blackness continues to belong to the definition of the original species of raven. Consequently,

(ii) All ravens ought to be black.

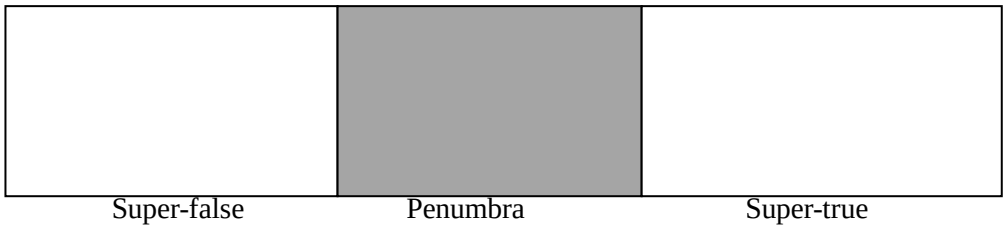
It seems then that this kind of vagueness between descriptive and normative claims is frequently found in ordinary language. Actually, Friedrich Waismann, one of the main members of the Vienna Circle which is usually known for what is called 'verificationism', proposed the open texture of empirical concepts, according to which 'the fact that in many cases there is no such thing as a conclusive verification is connected with the fact that most of our empirical concepts are not delimited in all possible directions' (Waismann 1951, pp. 119–120). His open-texture argument is akin to my point about the concept of 'raven'.[7] From my point of view, we could say that we treat empirical concepts as purely descriptive when we do not have to take extraordinary possibilities (like the appearance of pink raven) into account, whereas we are in a position to choose and declare a particular possibility as an only option in a normative way when we have to consider many possible treatments despite dealing with issues in an actual context. Waismann's open texture argument was initially concerned with theoretical knowledge, but was later developed in jurisprudence by H. L. A Hart. Hart argues that precedent or legislation works well in ordinary cases, but the open texture of things will become apparent when their application is in question (Hart 1961, p.124). One example Hart provides is the 'rule that no vehicle may be taken into the park' (Hart 1961, p.125). This general rule ordinarily works well. However, if we took an unenvisaged cases like 'a toy motor-car electrically propelled' (Hart 1961, p.126), we would be involved in an open texture,

[7] The linkage between my argument and Waismann's open texture argument was pointed out to me by an anonymous reviewer, for whose insightful observation I am most grateful.

where we would have to explicitly make a normative decision. As Hart's remark suggests, the open texture finally reaches normative judgement in spite of being proposed initially in the context of theoretical knowledge. This line of thought supports my argument on the vagueness between propositions (i) and (ii).

Another theoretical example is that of the ontological status of viruses. Are they living organisms or not? Should this question be empirically confirmed/falsified or regarded as a matter of definition, a concern limited to logical normativity? Answers to this question would fluctuate depending upon contexts and answerer. In this instance, the difference between whether a virus *is* or *ought to* be alive is vague. How should we understand such borderline cases, a product of the meta-vagueness of the distinction between descriptive and normative claims? One possible path is to adopt a form of supervaluationism, or to use a 'truth-value gap' approach by introducing the notion of a penumbra. The idea can be illustrated thusly:

Diagram 1



In this case, what matters in the evaluation of the truth is whether 'sentence X is descriptive' is true or not, or conversely, whether 'claim X is normative' is true or not. However, this idea can still be problematic. It is true that the sentence 'I am driving at a speed of 60km/h' is descriptive in any context in which it corresponds to the physical event. Thus, 'the sentence is descriptive' is also super-true in any precisifications (disregarding higher-order vagueness). However, could we understand borderline cases that may fall within the penumbra of this diagram? For instance, what of the case of cruelty in cutting a water flea in two? Consider the next sentence:

(iii) Cutting a water flea in two is cruel.

This sentence (iii) is probably true in a descriptive sense (when viewed through a microscope), but it also carries some (even though a very little) normative implication in that, qua cruelty, 'we ought not to do that'. In other words, what matters here is not whether sentence (iii) is descriptive or normative (i.e., whether 'sentence (iii) is descriptive' is neither true nor false [corresponding to the truth-value gap]), but *how* sentence (iii) is both descriptive and normative (i.e., how 'sentence (iii) is descriptive' is both true and false [corresponding to the truth-value glut])'. Thus, it is the degree of interaction that is of concern here; that is, the

extent of descriptivity and of normativity is in question. In this situation it is possible to see that descriptivity and normativity are mutually permeable, rather than following a penumbral pattern.

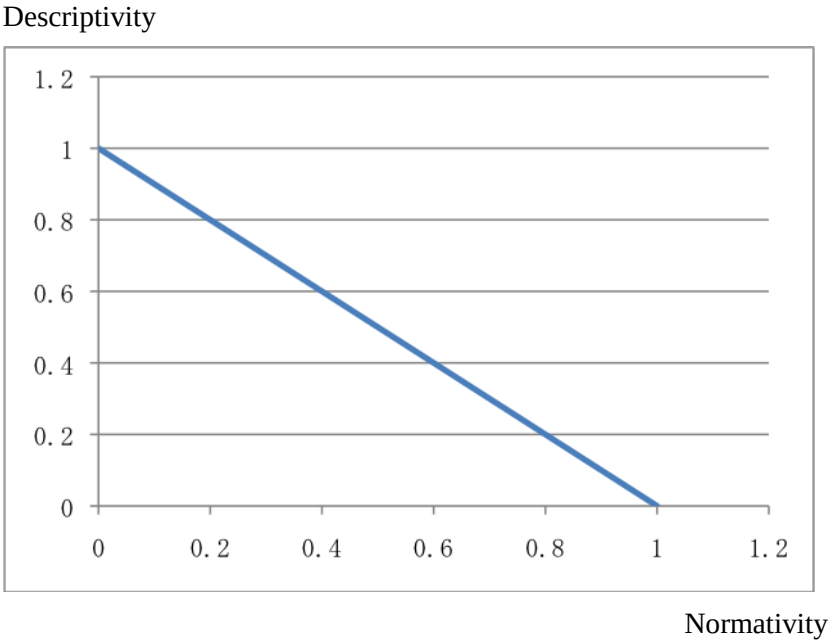
5 Degree by probability

Thus, in sentence (iii) both the implicit descriptive and normative claims interact, further supporting the idea that we should understand the interaction of the meta-vague predicates in terms of degrees. The degree of normativity in sentence (iii) is most likely less than the degree of normativity inherent in the next sentence (iv):

(iv) Cutting a human being in two is cruel.

In contrast, the degree of descriptivity in sentence (iii) may be more prominent than in sentence (iv), as we can observe the associated phenomenon corresponding to sentence (iii) more calmly than the case of sentence (iv), while the degree of normativity in (iv) is greater than that of (iii). How can this be understood? One possible answer is that the degrees of descriptivity and normativity complement one another. If we follow the standard custom of defining a 'degree' as a value found between the parameters of 0 and 1, we could plot the above circumstances in the form of a basic linear function (where the horizontal axis is the degree of normativity and the vertical axis that of descriptivity):

Diagram 2



This formulation may be valid, although perhaps too simple. Firstly, if the 'degree' displayed is to be measured in terms of probability (as will be done below), it satisfies Kolmogorov's axiom (see Howson and Urbach 1993, pp. 30–31). Secondly, and following from this, the significance of the Kripkenstein argument is strengthened by interpreting the notion of a 'rule' as being located at the point where the degree of normativity is 1 (correct) or 0 (incorrect). Each of these two points requires further discussion.

Firstly, it is necessary to discuss how the difference between a normative and descriptive claim can be measured in degrees. As has been mentioned, the primary difference can be shown in the contrast between what *ought to* be and what *is*. If *ought to* is interpreted generally (so that it includes all of the logical, social, moral, and legal normativities created by language), there seems to be a common axiom – a feeling of compulsory duty or force implicit in the acceptance of a normative statement. Perhaps, and historically speaking, this view corresponds to the Humean understanding of 'necessity' as 'a determination of mind' (see Hume 2000, T1.1.3.14.1). However, how might we measure this force? The theoretical formulation of a system of degrees is required. Consequently, I propose that the notion of (mainly interpersonal) subjective probability be used (or what might be termed 'degrees of belief') in the measurement of degrees of compulsory force. In the following example I draw upon a case of legal normativity, primarily because the case conspicuously highlights the character of normative force. A criminologist, Deryck Beyleveld, once proposed a utilitarian system with which to measure and evaluate the effect or deterrence of different punishments on specific criminal activities. His idea, broadly, was that the offence rate will inversely vary with both the likelihood of a sanction being imposed for committing a specific offence, and the severity of the sanction (Beyleveld 1979, p. 217).

Put differently, Beyleverld's idea can be understood as the claim that the psychological force that deters us from violating legal norms can be measured in terms of what might be called the 'expected severity of sanctions'. The (mainly interpersonal,) subjective severity of the expected sanction (abbreviated as SS) is multiplied by the (mainly interpersonal,) subjective, conditional probability of the sanction being imposed should the norm be violated (PS). The generalised application of this idea may serve as an example of how the degree of normativity concerning a statement or action can be measured, by emphasising the probability or severity of the associate sanction.[8] As a result, the degree of normativity (abbreviated as DN) can be expressed as a function of the expected severity of the sanction that would be imposed should a particular norm be violated. In this example, the subjective severity of the expected sanction is defined between the variables of 0 and 1, where 1 is the severest sanction

8 Philip Pettit has also developed a framework for understanding social norms by appealing to the notion of sanction. He offers the concept of the 'the intangible hand', through which people may suffer a sanction as a result of their own actions. This sanction motivates people to observe social norms (Pettit 1999, p. 225).

possible, such as being executed or killed. If we take normativity to be essentially connected with rationality (as is usually thought), it does not seem strange to judge the degree of normativity and consequently, the degree of rationality, from negative reactions to instances of their violation. Indeed, Joseph Raz has clearly highlighted this using the notion of 'blame' as a sanction: 'Common are standards which tie irrationality to blame. One's beliefs are irrational when one is blameworthy for having them' (Raz 2000, p. 43). Thus, we can represent this idea in the following equation, supposing 'A' to be a relevant sentence or expression:

Degree of normativity

$$DN(A) = SS(A) \times PS(A)$$

The intrinsic characteristics and mutual permeability of descriptive and normative claims become immediately apparent when understanding their difference by degrees in this way[9]. First, it can be seen that normative claims involve descriptive concerns, because DN is constituted by a degree of belief, which itself must ultimately be based on descriptive facts.[10] Even when we actively observe normative rules, we must also passively accept some unavoidable (descriptive) facts, without which the interactive practice of observing normative claims cannot begin at all (see Hookway 2000, p. 73, note 8). In order to consciously observe legal rules, for instance, we must (through experience) be aware of their existence and application, and have specific understandings, fears, and concerns with the sanctions that are derivative of the rules should they be violated. We must, in short, know what a legal and criminal justice system *is* in order to *obey* it. All of these processes should then be counted as the product of the internalisation of innumerable descriptive facts.

From this, a crucial point can be drawn – circumstances simultaneously correlating to a degree of normativity represented by 1 and a degree of descriptivity correlating to 0 (i.e., [degree of normativity, degree of descriptivity] = [1, 0]) are essentially impossible. This indicates that the formulation of their difference as a simple linear function (as in diagram 2) is not viable. Normative and descriptive claims are not dichotomously exclusive, nor can they satisfy Kolmogorov's axiom, even if they are frequently and rightly seen as in contrast to one another.

Secondly, as has been shown, the possibility remains that normative rules can be violated. Yet, this does not entail that DN(A) = 1 for any A is logically impossible. If the severest sanction (being killed) is always imposed in the case of a particular action or

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[9] Regarding the degree of descriptivity, I will discuss it further in section 7 below.

10 As far as the degree of difference between a normative and descriptive claim can be defined in terms of degrees of belief, the Bayesian approach (see Howson and Urbach 1993) may allow for dynamic interactions or updates, such as by interfacing with the social environment. This would require further exploration.

statement, the DN could possibly be 1 (presupposing that death is the most feared sanction). However, that does not mean we cannot violate the norm, at least physically, although we should then resign ourselves to the corresponding sanction. Peter Railton, for instance, has introduced this idea with regard to the freedom to violate rules or laws:

> The *must* here is not the *must* of something irresistible – the moral law is normatively, not actually, 'inviolable' … the normative domain must be a domain of freedom as well as 'bindingness'. (Railton 2000, p. 3)

Actually, for instance, it is considered a legal norm to observe the traffic law. Nonetheless, it can be broken, but then we must accept that there will probably be a consequence or sanction. The same is true of logical norms or rules like the law of non-contradiction. We ought to follow the norm to communicate with others, but it can be violated. However, our violation of the norm would cause other people to doubt our intelligence, which might negatively influence their evaluation of our personality. This should be a kind of social sanction[11].

This second point could lead to an interesting problem concerning the Kripkenstein argument. Kripke frequently presumes that the 'plus' function, rather than the 'quus' function, should be normatively justified. However, it need not be so. The 'quus' function may hold true *simpliciter*, in which our counterintuitive impression of the 'quus' function may be explained or understood in terms of the low degree of normativity it holds in relation to the (far more established) 'plus' function. As far as this theoretical sense, which presupposes a degree of difference, is concerned, Kripke's argument is viable. The 'plus' function should be more (although not entirely) justifiable than the 'quus' function simply because of difference in their degrees of normativity. Even so, we may choose to ignore the 'plus' function and adopt the 'quus' function, although in such a case we should accept some kinds of sanction (most probably a social sanction, such as being shunned). In other words, there is no true justification in a strict sense. My argument merely shows that 'plus' sounds relatively more justifiable than 'quus' based on empirical grounds using our subjective probability. If the environment around us were drastically different, it would be possible for 'quus' to sound

[11] I gave a talk on one part of my idea about DN in connection with the problem of causation by absence on 3 April 2015 at the 89th Annual Meeting of the American Philosophical Association Pacific Division, which was held in Vancouver of Canada. Ernest Sosa kindly gave me some comments, in which he asked how my idea of DN could treat the case of people lacking an ability to obey a norm. That is a quite significant question. For the time being, as long as I formulate that both SS and PS are measured from a mainly interpersonal point of view, SS should be evaluated with a consideration about people lacking an ability to obey a norm. But, this requires further clarification in detail, which is one of my future tasks.

more justifiable than ‘plus’. As a result, the notion of justification has to be applied very loosely when the DN is near 1, depending on our assignment of subjective probability. This kind of justification might be expressed as ‘being the most forced to be accepted’.

6 Pure normativity collapsed

The argument that has been made thus far suggests that the rule-following paradox may lead to an analytic stalemate so long as the clear-cut dichotomy between descriptive and normative claims is maintained despite the numerous of objections and counterexamples that have been mentioned. Consequently, we should be aware that there are no purely normative claims (such as would be needed to entirely justify only the ‘plus’ function) that can exist completely independently of any descriptive facts. In tracing out the parameters of this stalemate, it is necessary to revisit the Kripkenstein argument once more, to explore the consequences of seeking purely normative claims. In this example, the Kripkenstein argument will be applied to itself.

Could we say that we *ought to* deduce the consequent ‘Q’ from the premise ‘P ⊃ Q’ and ‘P’ according to *modus ponens*? This would of course be valid in normal circumstances. However, the Kripkenstein paradox casts doubt on this statement’s validity because even though we may have used the logical connectives ‘⊃’ and ‘and’ in the past (and so long as they maintain logical validity), the *possibility* remains that in reality we have used other logical functions, such as ‘⊃*’ and ‘and*’. As a result, we might derive a different consequent, such as ‘~Q’, which may be entailed by premises ‘P ⊃ Q’ and ‘P’. This example corresponds to the contrast between ‘plus’ and ‘quus’. Then, how should we understand the normative justification, if even logical normativity, which is truly the most fundamental, can be brought into doubt? This is a desperate state of affairs. As long as we accept the Kripkenstein argument in a literal sense and admit a ‘quus-like’ possibility on a par with our commonsense ideas like ‘plus’, we have no choice but to be involved in such a hopelessly chaotic situation. As a result, as far as we take the Kripkenstein paradox seriously, presupposing that the notion of justification applies only in a strict sense, it would be completely impossible to justify anything.

In contrast to the Kripkenstein paradox, Goodman’s grue paradox may seem more plausible because although he is concerned with issues of justification the concepts are not applied so strictly. In Goodman’s argument, the concept of justification remains connected to descriptive facts or our practices of inference (which can themselves be presumed to be descriptive facts). As he argues,

> Rules and particular inferences … are justified by being brought into agreement with each other. A rule is amended if it yields an inference we are unwilling to accept; an inference is rejected if it violates a rule we are unwilling to amend. The process of justification is the delicate one of making mutual adjustments between rules and

> accepted inferences; and in the agreement achieved lies the only justification needed for either. (Goodman 1983, p. 64)

Unlike the Kripkenstein paradox, Goodman's grue paradox does *not at all* depend upon a clear-cut dichotomy between descriptive and normative claims, although, of course, a broad distinction between the two categories is presupposed. Goodman, rather, emphasises the 'mutual adjustment' possible between the rules and inferences. This also supports the idea of a 'mutual permeability', as previously pointed out. This is the intrinsic difference between the Kripkenstein and Goodman paradoxes.

Moreover, Goodman's formulation of the grue paradox has little to do with purely normative claims. By way of example, even if all the emeralds we have hereto seen are green, this does not imply that we ought to expect that the next emerald will be green. The same is true of the predicate 'grue'. What is important in Goodman's paradox is which propositional hypothesis is more probable regarding the emerald's colour, and not whether we ought to conclude if it is true or not. Accordingly, Ian Hacking has described Goodman's claims as a 'pragmatic vision of justification' (Hacking 1993, p. 274). Indeed, it is obvious that Goodman's primarily focus is on inductive rather than normative justifications. As a result, the following of the rule, without admitting any exceptions (as in logical normativity), is irrelevant from the outset. It is a matter of degree.[12]

7 Descriptivity and the degree of unchangeability

Descriptive facts also need to be examined, particularly in relation to the Kripkenstein argument. As has been shown, the Kripkenstein argument concerning the rule-following paradox, at least on the surface, presupposes a clear-cut distinction between descriptive facts and their normative justification. However, in addition to showing that this distinction is itself not well founded, it seems that (in the paradox) descriptive facts may themselves be dubious. In a word, descriptive facts presented in the Kripkenstein paradox seem to be brute facts (as Searle once called them, in contrast to institutional facts (Searle 1969, pp. 50–53)) that can in principle be accurately and uniquely identified in an objective way. Their use, however, may be doubted, as can be simply shown. Any descriptive fact is only made meaningful through its linguistic articulation. As a result, those facts depend upon the meaning of words. Those words, however, are employed according to a normative framework that determines how they ought to be used, by such conventions as grammar and spelling. Indeed, Railton has declared that '*Spelling* is a normative concept' (Railton 2000, p. 4). As a result, and at a fundamental level, descriptive facts are intrinsically inseparable from issues of normativity, unless the existence of brute facts, without any linguistic mediation, can be established (which seems to

12 Sober (1994) provides an actual example of analysing the grue paradox in terms of probability.

be a naïve form of ontological realism).

Returning to the example of 'plus', Kripke argues that 'there is no fact about me that distinguishes between my meaning plus and my meaning quus' (Kripke 1982, p. 21). Nevertheless, he does acknowledge that there remains the (often hidden) fact that I still meant to say something, irrespective of whether it was 'plus' or 'quus'. However, should we not also take into account the possibility that the verb 'mean' is itself also involved in the rule-following paradox? Do we not need to consider, for example, the possibility that I may have meant 'nean' (which may denote antonyms for objects in a sentence that has never been used)? If such a possibility is to be seriously considered, the claim may fall into self-referential, infinite regress. Consequently, this possibility suggests that the Kripkenstein paradox may never be completed as, after all, the most basic word, 'mean', falls into the paradox, requiring the paradox to continue. As a result, it must be impossible to formalize the paradox itself in a fixed way.

This concern could also apply to the solution that Kripke provides for the rule-following paradox. As is well known, Kripke suggests the solution from a communitarian point of view:

> Any individual who claims to have mastered the concept of addition will be judged by the community to have done so if his particular responses agree with those of the community in enough cases … (and if his 'wrong' answers are not often *bizarrely* wrong, as in '5' for '68 + 57', but seem to agree with ours in *procedure*, even when he makes a 'computational mistake'). (Kripke 1982, pp. 91–92)

In response to this remark, two questions naturally arise. How can we confirm 'if his particular responses agree with those of the community'? Furthermore, how can or should we accept that the individual 'has mastered the concept of addition'? In point of fact, Kripke himself asks this question, and explains the concern by making an appeal to sensations:

> Since … the adult's confirmation whether he agrees with the child's avowal is based on the adult's observation of the child's behavior and circumstances, the fact that such behavior and circumstances characteristic of pain exist is essential in this case to the working of Wittgenstein's sceptical solution … If the individual generally makes his avowals under the right such circumstances, others will say of him that he has mastered the appropriate expression. (Kripke 1982, p. 100)

This may be an interesting way of solving the rule-following paradox. However, it seems to also evade the core problem highlighted by the paradox, and thus does not offer an analytically useful solution. Indeed, this solution appeals to the process of observation (which is perhaps a fact in itself), which intrinsically includes many predicates about perceptions, such as 'seeing', 'hearing', and so on, as long as Kripke focuses on the linguistic aspect of

observation by remarking ‘others will say of him’. Without those predicates of perception, the solution would not be attainable. What should we make of these predicates? Would the possibility of the rule-following paradox arise again in the use of the predicates? It seems that the paradox must permeate what we suppose to be brute facts themselves, once we have accepted a meaningful version of the paradox. As a result, so far as the Kripkenstein paradox is concerned, there really are no objective ways of escaping it apart from taking some normativity into account.[13]

To conclude the argument above, the Kripkenstein paradox would be self-contradictory because it (at least on the surface) presupposes a dichotomous, mutually exclusive distinction between descriptive and normative claims without considering the degrees of difference (or similarity) implicit in such claims. The Kripkenstein argument structurally discusses ‘pure normativity’ completely independent of descriptivity, so that it falls into serious difficulty of formulating itself in a fixed way. However, normativity and descriptivity, in reality, permeate each other to varying degrees, such that a ‘pure’ form of either is impossible. What is of concern, therefore, is *how* those degrees should be measured. This question has already been discussed with reference to the degree of normativity (DN). What, however, of the degree of descriptivity (DD)? It seems that the degree of difference between normative and descriptive claims is composed of a peculiar, countervailing asymmetry. There is a contrast between them, as I have mentioned, even though a clear-cut, dichotomous distinction should be denied. It seems that, generally speaking, we can interpret these two meta-predicates as bound by a correspondence between *expressions* (such as sentences, rules, and laws) and *phenomena* (such as states of affairs, utterances, or activities). The asymmetrical relationship could be understood by considering how one would deal with the difference that arises out of a conflict between the two resulting moments:

In case there is a conflict between *expressions* and *phenomena*,

13 That descriptive facts must involve some kind of normative claim can also be seen in other cases. For instance, if we connect the notion of normativity with that of rationality, as Raz discussed, we have to consider normativity as seriously concerning the rationality of every epistemic belief. As Raz notes, ‘When studying reasons we study normative aspects of the world … Our ability to reason is central to our rationality in all its manifestations, that is regarding reasons for belief, action, emotion, or anything else’ (Raz 2000, p. 43). If we understand the notion of normativity as a kind of guide or map drawn from our active deliberations on theoretical questions, as well as practical ones (such as Hookway developed), then normativity would conceptually permeate any description: ‘We can examine the normative standards that guide us when we try to carry out theoretical deliberations and conduct inquiries. These norms will guide us in formulating cognitive goals and selecting methods of deliberation or inquiry to employ in pursuit of them’ (Hookway 2000, pp. 60–61).

Descriptivity: *Expressions* should be corrected, while *phenomena* are unchangeable.
Normativity: *Phenomena* should be sanctioned, while *expressions* are retained.

The idea of contrast or asymmetry may roughly correspond with that of 'direction of fit', as John Searle once argued. Searle posits that 'belief' has the word-to-world direction of fit, while 'desire' or 'intention' has the world-to-word direction of fit (see Searle 1979, p. 3ff). These correspond to descriptivity and normativity respectively. If this formulation is correct, then it is not difficult to measure the degree of descriptivity (DD). In other words, we can measure DD by referring to the subjective probability concerning the 'unchangeability' of the phenomenon under question. For example, suppose that a particularly heavy storm washed over Japan on a particular day in 2013. We may firmly believe that this phenomenon has already occurred, so that it cannot be undone or changed. It has simply occurred. As a result, if someone claims that a heavy storm did not wash over Japan at that specific time and place, one can and should correct them. In this case, value 1 can be applied to the DD of the sentence, 'the heavy storm washed over Japan on that day in 2013', although even here there are implicit normative elements, especially with regard to the usage of the concept of 'Japan' and 'storm', not to mention spelling and grammar conventions. These are simply unavoidable, especially considering that normativity and descriptivity both permeate and are compatible with one another. These points are applied even to cases concerning the future phenomena, as the 'unchangeability' in question is based on our subjective probability, where we will assign probability 1 to future events which we are certain will occur (e.g., the sunrise tomorrow). Of course, purely contingent future events will be assigned lower degrees of unchangeability.

Moreover, in re-examining the borderline example of 'cruel', 'cutting a human being in two is cruel' is intended to be a descriptive statement as long as we highlight the physical aspects associated with the phenomenon. As far as those aspects are concerned, in case there is a conflict between *expressions* and *phenomena* (e.g., a case wherein we say 'cutting a human being in two is NOT cruel'), the statement should be corrected, whereas the phenomenon is itself unchanged. However, the statement is not simply descriptive. Unlike in the example of the storm, the status of the two meta-predicates is more likely to fluctuate in the cruelty case. In other words, this case is no less normative than it is descriptive. If someone performed an action of cutting a human flesh in two, the act could itself be judged and blamed. As a result, the phenomenon or action (cutting human flesh) is what is sanctioned. Here the normative rule, 'we ought not to cut a human being in two' is meant by '*expression*' in 'the case where there is a conflict between *expressions* and *phenomena*'.

In any case, and in following these considerations, we can propose the next formula in the form of a conditional probability, signified by DD (supposing 'A' to be a relevant sentence or expression as in the case of DN, and PrA to be the phenomena relevant to 'A'):

Degree of descriptivity

$$DD(A) = P(\text{PrA is unchangeable} \mid \text{A differs from PrA})$$

As a result, we have arrived at a place where we can understand the difference between *is* and *ought to* more appropriately, by degrees of difference.

8 The possibility of an appropriate model

How then should we re-formulate the relationship between DN and DD to make an appropriate model of the relationship? My argument thus far could be summarised in the following two requirements.

1) The asymmetry between descriptivity and normativity must be reflected in an appropriate model, although a clear-cut, dichotomous distinction between those two meta-predicates should be denied. In other words, the appropriate model must reflect such a situation that the more DD becomes, the less DN becomes, and vice versa.
2) Neither pure descriptivity nor pure normativity can obtain. Namely, neither value of [DD, DN] = [1, 0] nor that of [DD, DN] = [0, 1] is possible.

Probably, the simplest model that satisfies those two requirements would be a linear function expressed by, for instance, DD + DN = a (> 1). However, this possibility does not seem plausible, for if we accepted it, then we should admit that the sum of DD and DN is *always* more than 1. This is not suitable as a model constituted by degrees between 0 and 1 of parameters (i.e., DD and DN) asymmetrical to each other and not realistic either. Recall sentence (iii) in section 4 above, that is, ‘Cutting a water flea in two is cruel’. Its DD would be much less than 1, as, if someone claims that cutting a water flea in two is *not* cruel, we are not fully convinced that sentence (iii) is still true (i.e., absolutely unchangeable). Furthermore, the DN of the sentence (iii) is rightly assessed as very low, perhaps near 0, although not exactly the same as 0. Actually, we can unwittingly cut water fleas when manipulating water in nature, but nobody is likely to blame us. Thus, in the case of sentence (iii), the sum of DD and DN is less than 1.

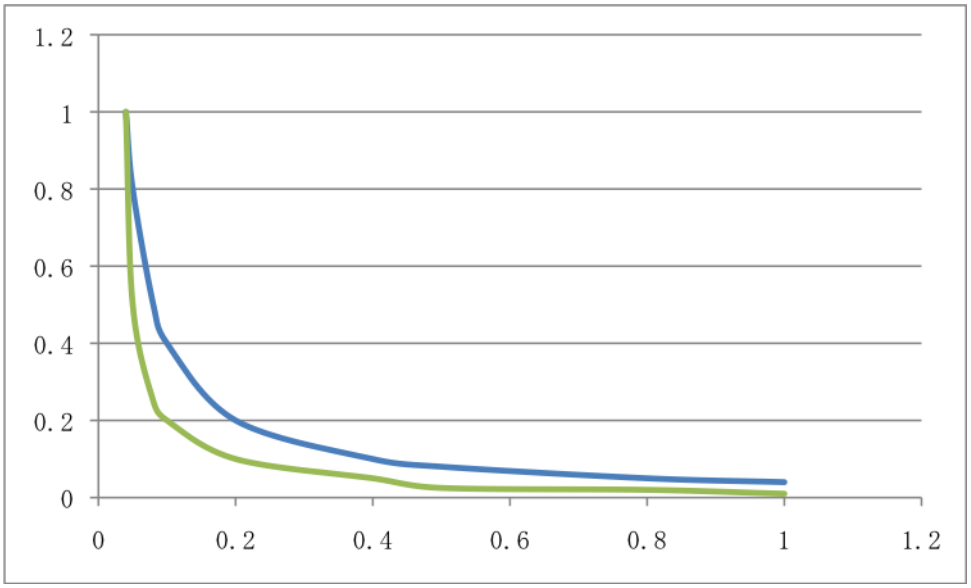
The next possible candidate that satisfies those two requirements is, probably, a hyperbola model, which could be formulaically understood thusly:

$$DN \times DD = k \quad (k \text{ is a constant, } 0 < k < 1)$$

This hyperbola model seems to meet those two requirements in a more refined way than the linear function model, as we can smoothly place cases like sentence (iii) in the hyperbola. The model could be graphically illustrated thus (hypothetically presupposing that cases with a degree of less than zero do not exist):

Diagram 3

Degree of descriptivity



Degree of normativity

We can easily recognize that there are some parts of this model where the sum of DD and DN is less than 1, corresponding to the case of sentence (iii). The diagram illustrates two curves, representing the difference values of *k* which correspond to (although not precisely stand for) the degree of truth (or, in other words, the degree of acceptability). For example, logical truths have the maximum value of *k* (here in the diagram 3 I tentatively presume that the maximum value of *k* is 0.04), whereas vague sentences like 'a man whose height is 169 cm is tall' have a lower *k* value. Consequently, if we were to temporarily adopt this model, a tentative answer could be given to Kripke's distinction between the 'plus' function and the 'quus' function. In point of fact, the 'plus' function is located close to the point where DN = 1, with almost the highest value of *k* (where its DD is not absolute zero), while the 'quus' function has a much lower DN and much lower value of *k* (even where its DD is relatively high). Within the scheme about DN, DD, and *k*, the 'plus' function is relatively more justifiable than the 'quus' function, as Kripke argues, even though we should not expect its true justification in a strict sense. Similar answers can be extended to some of the difficulties mentioned before such as the contrast between '⊃' and '⊃*' or between 'mean' and 'nean' through considering the difference of DN of each.

In addition, if we adopted the hyperbola model, we might deduce an interesting suggestion about the problem of vagueness in general. The hyperbola model seems to match the paraconsistent dialetheism (based upon the truth-value glut approach) concerning the problem of vagueness and the sorites paradox, especially with regard to the end zones around DD = 1 or DN = 1 in diagram 3, no matter how much value *k* is assigned, as the sum of DD and DN is more than 1 around those zones. However, if *k* is assigned a very low value, the sum of DD and DN could be less than 1 in the middle zone, where the hyperbola model could

match the paracomplete theory (based upon the truth-value gap approach) at least around the zone. In suggesting these, I am following the characteristics given by Field with regard to paraconsistent dialetheism and paracomplete theory (Field 2008, p. 364).[14] Although my argument is, for the time being, only concerned with meta-vagueness, I hope that this view will find a general applicability in broader debates concerning vagueness, suggesting that paraconsistency could be compatible with paracompleteness.

Admittedly, the hyperbola model, strictly speaking, is far from perfect, though it works as a springboard for discussion. For one thing, it is not at all clear how to determine the value of the constant *k* in relation to DD and DN,[15] even though DD and DN can be precisely measured in terms of subjective probability as argued thus far. This lack of clarity probably originates from my first strategy of incorporating three distinct kinds of degrees (DD, DN, and *k*) into a two-dimensional diagram. This point could be illustrated by comparing the next sentence (v) with sentences (iii) and (iv) above.

(v) Cutting a teddy bear in two is cruel.

It is instantly apparent that it is vague whether sentence (v) is true or not. This vagueness should correspond to how to determine *k* in my hyperbola model above. Obviously, the vagueness represented above is on a different level from that of arguing about whether the sentence (v) is descriptive or normative. This difference is exactly that between vagueness and meta-vagueness emphasized in section 2 above. Thus it is not entirely reasonable to represent this vagueness in the same plane with DD and DN. However, simultaneously, it still seems that the vagueness represented by sentence (v) is somehow interwoven with DD and DN. Consequently, in order to correctly analyse the situation, it would be helpful to develop a *three-dimensional* formalisation, which I am not able to carry out right now.

Secondly, and more fundamentally, I am not entirely convinced that the actual relationship between DD and DN could be traced as a kind of function in a mathematical

14 According to Field, paraconsistent dialetheism takes P(A) + P(¬A) to be greater than or equal to 1, while paracomplete theory takes P(A) + P(¬A) to be less than or equal to 1 (Field 2008, p. 364). I regard P(A) and P(¬A) as corresponding to DD and DN on the ground that there must be asymmetry between DD and DN. Of course, this is just a possibility, requiring more meticulous research. That is another subject for another paper.

15 This crucial drawback was correctly pointed out by my colleague, Richard Dietz. I appreciate his acute observation. Actually, an anonymous reviewer also raised a question concerning the value of *k*. Should there be any constraints on the possible values *k* may take? I have no firm view on this question. As for the diagram, I presume that the maximum value for *k* in the diagram 3 should be 0.04. At the end of the day, I regard this question as empirical in nature.

formalization. It seems we have to take into account the possibility of conducting empirical, statistical research tied up with experimental philosophy or psychology which might result in finding some irregular or unformalizable relationship between DD and DN actually obtains. This needs further investigation.

Nevertheless, I do not think that my argument thus far is insignificant. For instance, my claim on the mutual permeability of descriptivity and normativity could lead to a defence of Mill's famous view, resisting Moore's naturalistic fallacy argument by claiming that the notion of 'desired' seems to be based upon a tacit agreement of normative values – such as 'desirable' – as well as descriptive facts. Indeed, Raz has noted, 'Since we desire only what we think of as worth desiring, our desires are among our responses to perceived reasons' (Raz 2000, p. 42). This implication will be quite helpful, for example, in re-examining how a naturalistic analysis of moral issues should be evaluated. Additionally, my introduction of the concepts of DD and DN and how to measure them will compel the discussion forward toward a more thorough analysis of the intertwined relation between descriptive modes like 'desired' and normative modes like 'desirable'. My ideas on such concepts of degrees would be analytically applicable to many problems in philosophy. Indeed, the contrast between descriptivity and normativity is crucial to practical and political issues (e.g., territorial issues or legislation about the succession of thrones) as well as philosophical discussions. Thus I hope my view on DD and DN can inform and refine practical discussions in various fields.

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